Dr. Cristian Giardinà

Stochastic Processes 2WB08: problem set 1

Martingale definition and first properties

1. Review conditional expectations [Ross, Section 1.5]. In particular, prove the properties of conditional expectation given in the lecture. If you fill the need of a further understanding of conditional expectation, work out (with closed book!) examples 1.5(A), 1.5(B), 1.5(D).

2. From the book [Ross], problems 6.1, 6.2, 6.3, 6.4, 6.5, 6.6, 6.7.

3. Let U_i, $i \geq 1$, be i.i.d. uniformly distributed on $(0,1)$, and set

$$X_n := \prod_{i=1}^{n} U_i \qquad \text{and} \qquad M_n := 2^n X_n$$

for $n \geq 1$. Show that $(M_n)_{n \geq 1}$ is a martingale.