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Toegepaste kansrekening 2WS15
problem set 1

- Study Chapter 1 (all) of F.Den Hollander book "Large Deviations". Especially, pay attention to the proof of lower bound in the Cramer theorem (exponential tilting).
- Problem 1 Work out all details of the proof of Theorem I.3 (coin tossing) by using the Stirling formula $n! = n^n e^{-n}\sqrt{2\pi n}\left(1+\mathcal{O}\left(\frac{1}{n}\right)\right)$.
- Problem 2 Compute the rate function $I(z)$ for the large deviations of the sample mean $\frac{1}{n}S_n = \frac{1}{n}\sum_{i=1}^{n} X_i$, where $(X_i)_{i\in\mathbb{N}_0}$ is a sequence of i.i.d. random variables with the following distributions:

  1. $X_i \sim Poisson(\lambda)$
  2. $X_i \sim Exponential(\lambda)$
  3. $X_i \sim Normal(\mu, \sigma^2)$

  Answers:

  1.
$$I(z) = \begin{cases} z\log\left(\frac{z}{\lambda}\right) - z + \lambda & \text{for } z \geq 0 \\ +\infty & \text{for } z < 0 \end{cases}$$

  2.
$$I(z) = \begin{cases} \lambda z - \log(\lambda z) - 1 & \text{for } z > 0 \\ +\infty & \text{for } z \leq 0 \end{cases}$$

  3.
$$I(z) = \frac{(z-\mu)^2}{2\sigma^2} \qquad \text{for } z \in \mathbb{R}$$