Dr. Cristian Giardinà

Stochastic Processes 2WB08 problem set 2

Martingale definition and further properties. Azuma-Hoeffding inequality

- From the book [Ross] problem 6.8, 6.9.
- Let $(X_n)_{n\in\mathbb{N}_0}$ be a Markov chain with finite state space S and transition matrix p. Let $g : S \to \mathbb{R}$ be a function with the following property: there exists $\lambda > 0$ such that for all $i \in S$ the inequality
$$\sum_{j\in S} p(i,j)g(j) \le \lambda g(i)$$
holds. Prove that $(\lambda^{-n} g(X_n))_{n\ge 0}$ is a supermartingale.
- Let $(M_n)_{n\ge 0}$ be a strictly positive martingale. Show that $Z_n = -\ln M_n$ is a sub-martingale, provided the appropriate integrability condition holds true.
- From the book [Ross] problem 6.15, 6.16, 6.17, 6.18, 6.19, 6.20, 6.21.