

**Toegepaste kansrekening 2WS15**  
**problem set 2**

- Study Chapter 2, Sections 1-4, of F. Den Hollander book “Large Deviations”.

Let  $\rho$  be the uniform distribution on  $\Gamma = \{1, \dots, r\}$ .

- Problem 1. Give and explain an example of a realization of a sequence  $(X_i)$  of i.i.d. random variables with law  $\rho$ , such that a large deviation of the empirical measure occurs, but not of the empirical mean.
- Problem 2. Give and explain an example of two different measures  $\nu$  and  $\nu'$  on  $\Gamma$ , that have the same relative entropy with respect to  $\rho$ .
- Problem 3. Compute  $I_\rho(\nu)$  when  $\nu$  is the uniform distribution on a subset  $\Gamma'$  of  $\Gamma$  with  $|\Gamma'| = r'$ .

**Hints on problems 1 and 2:**

- Choose  $r = 3$ . What is the expected mean? Let  $(X_i)$  be such that no 2's occur. How can you choose the sequence such that the empirical mean is the same as the expected mean?
- Choose  $r = 2$ , and write out the expression for the relative entropy.

**Answers:**

- Choose  $r = 3$ , and let the sequence  $(X_i)$  have an equal number of 1's and 3's, but no 2's.
- Choose  $r = 2$ , and  $\nu_1 = \nu'_2 = p$ . Then  $H(\nu|\rho) = H(\nu'|\rho) = p \log 2p + (1 - p) \log 2(1 - p)$ .
- $I_\rho(\nu) = \log r/r'$ .