Screened Vortex Lattice Model with Disorder

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The three dimensional XY model with quenched random disorder and finite screening is studied. We argue that the system scales to model with $\lambda \simeq 0 \simeq T$ and the resulting effective model is studied numerically by defect energy scaling. In zero external field we find that there exists a true superconducting phase with a stiffness exponent $\theta \simeq +1.0$ for weak disorder. For low magnetic field and weak disorder, there is also a superconducting phase with $\theta \simeq +1.0$ which we conjecture is a Bragg glass. For larger disorder or applied field, there is a non superconducting phase with $\theta \simeq -1.0$. We estimate the critical external field whose value is consistent with experiment.

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The phase diagram of type II superconductors in an external field has been the subject of intense theoretical and experimental investigation [1]. After the discovery of high-T_c materials, the role of both thermal and disorder induced fluctuations has been reconsidered, revealing many new interesting phenomena. In clean systems, it was realized [2] that, with increasing temperature, the Abrikosov lattice melts into a vortex liquid via a thermally induced first-order transition. Experiments performed on thermodynamic quantities such as magnetization [3] and specific heat [4] confirmed the first order nature of the melting transition in YBCO and BSSCO materials. More recent studies of the clean unscreened system in an external field [5] have shown that the low temperature vortex lattice melts at $T = T_M$ to a liquid of rigid flux lines and at $T_L \geq T_M$ the lines become entangled and vortex loops proliferate. Very recently, there have been studies [6, 7, 8] of a random $3D$ XY model wthout screening. These claim to study the glass transition in a superconductor but do not have the vital screening and obtain results which are inconsistent with each other so their relevance to real systems is unclear.

In this Letter, we study the stability of superconductivity in the presence of point disorder and an applied magnetic field by computing numerically the stiffness exponent θ. We argue that the effective screening length λ and temperature T scale to zero at very long length scales so that a model with $\lambda \simeq 0 \simeq T$, which is amenable to simulation, is a physically reasonable model of a disordered superconductor at low T. Using this effective model, we can identify the field and disorder driven transition [9] from a superconducting to non-superconducting phase as the field or disorder is increased and conjecture that the former is a Bragg glass [10]. The nature of the large disorder phase is still controversial: a viscous non-superconducting pinned vortex liquid [11] or a superconducting vortex glass [12].

The ingredients necessary to describe a typical high-T_c superconductor in a field are (i) pinning of flux or vortex lines and loops by random point impurities and (ii) weak screening of the interactions between vortices. We argue that a model containing these essential ingredients is a three dimensional XY model on a simple cubic lattice with quenched random phase shifts. In the vortex representation, the Hamiltonian is [13, 14, 15]

$$H = \frac{1}{2}\sum_{i,j} G(i,j)(\boldsymbol{J}_i - \boldsymbol{b}_i)\cdot(\boldsymbol{J}_j - \boldsymbol{b}_j) \tag{1}$$

We may ignore boundary terms which, at least in the best twist approach, vanish by a proper choice of global twists [16]. The dynamical variables are the integer valued vorticities $\boldsymbol{J}_i$ on the links of the dual lattice and subject to the local constraint $(\nabla\cdot\boldsymbol{J})_i = 0$ at every site i. The $\boldsymbol{b}_i$ are quenched random fluxes on the dual lattice which are obtained from the circulation of the quenched vector potential $\boldsymbol{A}$ and by adding a uniform external field h in the $\hat{\boldsymbol{z}}$ direction

$$\boldsymbol{b}_i = \frac{1}{2\pi}[\nabla\times\boldsymbol{A}]_i + h\hat{\boldsymbol{z}} \tag{2}$$

Here $h \equiv Ba_0^2/\Phi_0$ is the mean flux per elementary plaquette normal to the applied field $B\hat{\mathbf{z}}$, a_0 is the underlying lattice spacing and $\Phi_0 = 2\times 10^{-7}$ Gauss·cm^2 is the flux quantum. The vector potential $A_{\mu i}$ with $\mu = x, y, z$ is independently uniformly distributed $A_{\mu i} \in [0, 2\pi\alpha)$ with $0 \leq \alpha \leq 1$ and is defined on the bonds of the original lattice. The *disorder strength* α interpolates between two well known limits, the clean system, $\alpha = 0$, and the maximally disordered, $\alpha = 1$, gauge glass. By construction, the fields $\boldsymbol{b}_i$ satisfy the divergenceless condition $(\nabla\cdot\boldsymbol{b})_i = 0$ on every site. $G(i,j)$ is the screened lattice Green's function with dimensionless screening length λ in units of a_0

$$G(i,j) = \frac{(2\pi)^2}{L^3}\sum_{\mathbf{k}}\frac{\exp[i\boldsymbol{k}\cdot(\boldsymbol{r}_i - \boldsymbol{r}_j)]}{2\sum_\mu(1-\cos k_\mu) + \lambda^{-2}} \tag{3}$$

where $\boldsymbol{r}_i = (x_i, y_i, z_i)$ is the i-th site on the dual lattice and $k_\mu = 2\pi n_\mu/L$, with $\mu = x, y, z$ and $n_\mu = (1,\ldots,L)$. The vortex system defined by Eqs. (1), (2) and (3) is a vortex lattice of spacing $h^{-1/2}$ for $\alpha = 0$ in the absence of disorder. Turning on disorder distorts the lattice and pins it at a favorable position in the underlying substrate lattice, as is clear from Eq. (1), but does not destroy the underlying periodicity of the vortex structure [10]. Real cuprate superconductors are very anisotropic with $\Gamma = \lambda_z/\lambda_{x,y} \approx 5$ for YBCO and $\Gamma \approx 100$ for BSSCO [1] where we have chosen the $\hat{\boldsymbol{z}}$ direction as the c axis and $\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}}$ directions as the a, b axes. We note that when $\lambda_\mu \ll 1$, the vortex-vortex interactions become isotropic [17] and, in this limit, our isotropic model is realistic.

The final and most important part of our argument is to justify the validity of the strong screening limit for an extreme type II superconductor where the bare screening length $\infty > \lambda \gg 1$ is large but finite. ¿From Eq. (3), it is clear that λ scales like a length so that scaling $a_0 \to e^l a_0$ induces the scaling $\lambda \to e^{-l}\lambda$. Thus, at long length scales which are relevant to the weakly disordered system of interest, the effective screening length $\lambda \ll 1$. In this strong screening limit, $G(i,j) = (2\pi\lambda)^2\delta_{ij} + \mathcal{O}(\lambda^4)$ which yields

$$\frac{H}{(2\pi\lambda)^2} = \frac{1}{2}\sum_i(\boldsymbol{J}_i - \boldsymbol{b}_i)^2 + \mathcal{O}(\lambda^2) \tag{4}$$

This local form of Eq. (4) on a cubic lattice can be regarded as a model for the long length scale properties of a disordered superconductor and can be studied numerically by very efficient combinatorial optimization algorithms [18, 19] on large systems. Any nonlocal terms such as vortex - vortex interactions or boundary terms [16, 20] in Eq. (1) render such algorithms useless. Of course, screening is weak, $\lambda \gg 1$, in a real type II superconductor and one may question the relevance of a system described by Eq. (4). Since our interest is in the superconductivity and in the vortex lattice structure as a secondary effect, we argue that Eq. (4) is an adequate description of a superconductor in an external field with any finite screening. Numerical simulations [16, 20, 21] of Eq. (1) indicate that the exponent θ has the same value for *any* $\lambda < \infty$. We find this slightly surprising as decreasing λ transforms a type II into a type I superconductor and the independence of θ on λ seems to imply that these do not differ in any essential way at small T. Additional justification for the relevance of the model of Eq. (4) at $T = 0$ is that T is an irrelevant variable [10] scaling to zero in the superconducting region. This gives some justification for the computationally accessible model of Eq. (4) and for our scaling arguments.

To investigate whether a transition occurs, we use defect energy scaling. In this approach, one computes the energy $\Delta E(L)$ of a defect in a system of linear size L and fits to the scaling *ansatz*

$$\langle \Delta E(L)\rangle \sim L^\theta \tag{5}$$

where θ is the stiffness exponent and $\langle\cdots\rangle$ denotes an average over disorder. The sign of θ distinguishes two regimes: if $\theta > 0$, inserting a defect costs an infinite energy in the thermodynamic limit and the system will be ordered at sufficiently small finite T. Conversely, if $\theta < 0$, large domains cost little energy and, at any $T > 0$, superconductivity will be destroyed. To calculate the defect energy we employ the method proposed by Kisker and Rieger [22], who restated the problem of finding the ground state for Hamiltonian (4) in terms of a minimum-cost-flow problem [18],where the cost functions are precisely given by $c_i(\boldsymbol{J}_i) = (\boldsymbol{J}_i - \boldsymbol{b}_i)^2/2$. This method makes use of the successive shortest path algorithm (SSPA) [19] to find the ground state configuration $\{\boldsymbol{J}^0\}$ for each realization of disorder. The global flux $\boldsymbol{f}$ associated with this configuration is given by

$$\boldsymbol{f} = \frac{1}{L}\sum_i \boldsymbol{J}_i^0 \tag{6}$$

The elementary low energy excitation configuration $\{\boldsymbol{J}^1\}$ is obtained by gradually decreasing all costs in, say, the z direction until the global flux f_z jumps by one, $f_z \to f_z + 1$. The lowest energy excitation will be a global vortex loop encircling the $3D$ torus in the z direction. The defect energy is then obtained by $\Delta E = E(\{\boldsymbol{J}^1\}) - E(\{\boldsymbol{J}^0\})$. A more conventional way of determining the defect energy is to calculate the energy difference between periodic and antiperiodic boundary conditions, which amounts to adding a global twist of π along one spatial direction [16]. In our case we can ignore the boundary terms because the global loop corresponds to a twist of 2π, which has no effect as the original Hamiltonian is invariant under a discrete gauge transformation modulo 2π. Remarkably large system sizes can be treated by applying the SSPA, while conventional methods such as repeated quenching or simulated annealing are much less efficient. In this work we study $L \times L \times L$ systems with $L \leq 40$ for different values of α in the range $0 \leq \alpha \leq 1$ and magnetic field in the range $0 \leq h \leq 0.25$. The number of realizations of the random bonds varies from 500 for 40^3 systems up to 10^4 for the smallest ones. There is no upper limit on L.

The zero field case has been studied in detail [23] and we repeated the simulation to check our algorithm. The results are summarized in Fig. (1) for several values of α and they are essentially identical to those of Ref. [23]. For weak disorder, $\alpha < \alpha_c \simeq 0.495$, we obtain $\theta \simeq +1.0$ indicating a superconducting phase for $T > 0$. For strong disorder, $\alpha > \alpha_c$, the stiffness exponent $\theta \simeq -1.0$ indicating a non superconducting phase for $T > 0$.

In an applied field type II superconductors have a fixed density of vortex lines which, with weak disorder, form a distorted Abrikosov lattice or Bragg glass [10] at small T. Increasing the field effectively increases the disorder and the Bragg glass phase transforms into an entangled array of vortex lines with no remnant of translational order. The model of Eq. (1) contains the standard description

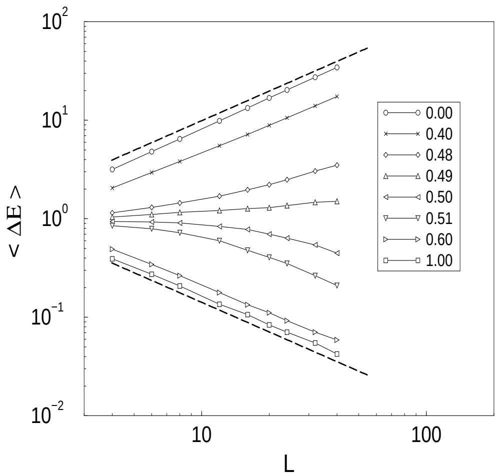

FIG. 1: Size L dependence of domain wall energy $\langle \Delta E \rangle$ in zero external field (log-log plot). The legend shows the magnitude of the disorder strength. Solid lines are guides for the eyes, dashed lines with slopes ± 1 are drawn for reference.

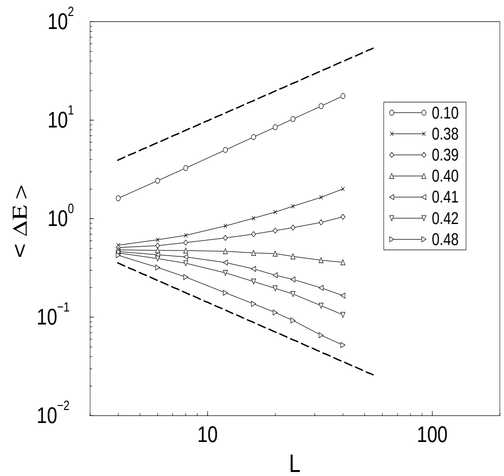

FIG. 2: L dependence of domain wall energy $\langle \Delta E \rangle$ for finite external field $h = 0.25$. Varying the disorder strength (see legend) changes the sign of the stiffness exponent θ. Dashed lines have slopes ± 1, while solid lines are guides for the eye.

of a vortex lattice as an elastic periodic solid in a random pinning potential when $\lambda > 0$. The essential superconducting properties seem to be unchanged as $\lambda \to 0$ but the interaction of the flux lines determining the periodic lattice structure vanishes when $\lambda = 0$. Our strategy is to impose the required periodic vortex lattice structure by imposing a periodic array of columns of bonds along $\hat{\mathbf{z}}$ which favor flux lines on these columns. We make the plausible conjecture that this procedure retains the essential physics of a disordered superconductor at $T \simeq 0$ and it is the only system which can be simulated with large size L. To compute the defect energy $\Delta E(L)$ it is essential to impose periodic boundary conditions in all directions and imposing a fixed number of flux lines by the source/target method [18] is not applicable.

We implement the periodic lattice potential by first adding a uniform $h = Ba_0^2/\Phi_0$ to all $\hat{\mathbf{z}}$ bonds. To ensure that the vortex density does correspond to this h, we introduce a potential Δ on the bonds of the expected vortex lattice by $h \to h + \Delta$ in Eq. (4). The costs of the flux lines are decreased by increasing Δ until the matching is satisfied with one flux line per favorable bond. We thus obtain a ground state of energy $E_0(L)$ with the required number $N = hL^2$ of flux lines. The defect energy $E_D(L)$ is obtained in the standard way [22, 24] by reducing costs until an extra flux loop is induced. This is the simplest way of introducing the essential periodic part of the random pinning potential on the flux lines [10]. It is interesting to note that our model is essentially that of many elastic lines in a periodic potential [25] so the transition at $\alpha_c(h)$ is a transition from a superconducting state to a state which is both rough and entangled. This is in contrast to a superconductor in a field with no disorder where the vortex lattice first loses translational order and becomes a liquid of rigid lines and these lines become entangled at larger field or temperature [5].

In Fig. (2) we show the behavior of the defect energy with system size L for a fixed value of external field $h = 1/4$ for different disorder strengths α. It is more convenient to fix the field and vary αbecause the period of the vortex lattice must be commensurate with the system size L which allows only restricted values of $h = NL^{-2}$ with N, L integers. At small disorder, $\alpha < 0.40$, we observe a positive stiffness exponent, which asymptotically tends to $\theta = 1$. Increasing the disorder, $\alpha > 0.40$, we find that the defect energy decreases with L and for large L, $\langle \Delta E(L) \rangle \sim L^{-1}$. There is a critical value $\alpha_c(h = 0.25) \simeq 0.40$ separating an ordered from a disordered phase. Fig. (3) shows typical ground state configurations of the system. Below the critical disorder, Fig. (3) left, the lowest energy configuration forms an almost perfect vortex lattice, while above the critical disorder, Fig. (3) right, the lines are rough and entangled [25], as expected for a phase with proliferation of dislocations. A similar analysis for different values of h shows that $\alpha_c(h)$ decreases monotonically as h increases as shown in Fig. (4). We estimate the critical value of the external field as $h_c = \mathcal{O}(1)$ where $h \equiv Ba_0^2/\Phi_0$ is the mean flux per plaquette. Here B is the actual field, a_0 is the lattice spacing and $\Phi_0 = 2 \times 10^{-7}$ Gauss$\cdot$cm^2 is the flux quantum. In the strong screening limit, $a_0 \sim \lambda$, whose typical value in high-T_c superconductors is $\lambda \simeq 10^{-5}$ cm. This yields an estimate for the critical field $B_c \simeq 10^3$ Gauss. This is to be compared with the typical experimental value of $B_c \simeq 500$ Gauss in BSSCO [9]. However, this should also hold for YBCO which has a comparable λ at $T = 0$ [1]

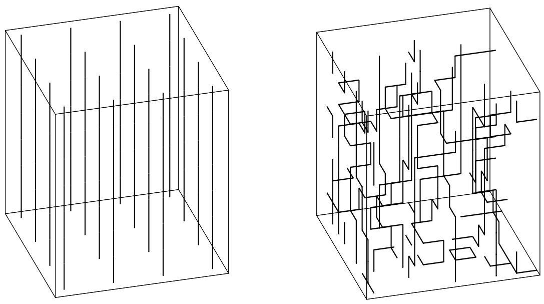

FIG. 3: Ground states for $h = 0.25$ in the ordered phase, $\alpha = 0.10$ (left) and in the disordered phase, $\alpha = 0.48$ (right).

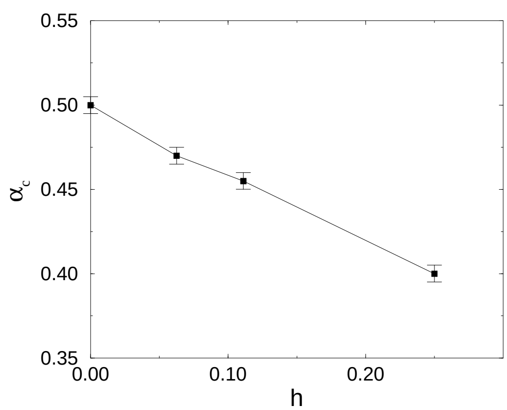

FIG. 4: Critical disorder strength $\alpha_c(h)$ for $0 \leq h \leq 0.25$.

and our estimate of B_c depends on λ only. This seems to disagree with experimental estimates of $B_c \sim 10^4$ Gauss [26] but the measurements are for $T > 50^o K$.

In this Letter we have studied the strongly screened vortex glass in the presence of disorder and, for the first time, have successfully implemented a defect energy scaling study of the stability of superconductivity as indicated by the stiffness exponent θ in an external field using *periodic* boundary conditions. It would be interesting to apply this method to probe directly the behavior of the model allowing for dislocations in the vortex lattice by applying appropriate boundary conditions [27]. It would also be interesting to treat a more realistic model with finite screening and to show more convincingly that the $T = \lambda = 0$ limit studied in this Letter is the stable renormalization group fixed point of a disordered superconductor at low temperature.

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